Abstract—Hybrid precoding, a combination of analog and digital precoding, is an attempt to reach a compromise between complexity and performance. By exploiting more than one RF chain, hybrid precoding enables a millimeter wave (mmWave) system to take advantage of both spatial multiplexing and beamforming gain. A major challenge with hybrid precoding is its configuration in wideband systems. The reason is that the analog beamforming weights are the same across the entire band. In this paper, we propose a frequency selective hybrid precoding technique for a mmWave orthogonal frequency division multiple access system. We propose an algorithm to jointly optimize the wideband analog beamformers and the per-subcarrier digital precoders. We compare an upper bound with our solution and show that our optimized approach achieves good performance in terms of beamforming gain in the sense that it minimizes the loss caused by allocating multiple users to different subcarriers with a limited number of radio frequency chains.

I. INTRODUCTION

Millimeter wave (mmWave) carrier frequencies offer the potential for high bandwidth communication channels in next generation cellular system [1]–[3]. One challenge in mmWave systems is obtaining sufficient received signal strength. The high carrier frequency provides a clue to the solution for this problem. The short wavelength in mmWave channels enables a large number of antennas to be packed into a small form factor and consequently offers high antenna array gain. Using a large number of antennas, however, causes another problem from a system design point-of-view. At frequencies less than 6 GHz, multiple antenna techniques such as MIMO or beamforming are usually performed digitally at baseband with each transmit antenna requiring its own dedicated radio frequency chain and digital-to-analog converter (DAC). At higher carrier frequencies and bandwidths, this complete digital implementation is challenging due to computational complexity and power consumption. For these reasons, state-of-the-art commercial mmWave systems like IEEE 802.11ad use analog beamforming with only one RF chain as an alternative to multi-stream MIMO with digital beamforming or precoding [4]. Unfortunately, analog beamforming has several drawbacks: 1) it supports only single-stream MIMO, 2) it does not allow multiuser spatial multiplexing, 3) it uses one beamformer for the entire frequency band.

Hybrid precoding attempts to resolve some of the disadvantages of analog beamforming. It allows multi-stream operation through the use of multiple banks of beamformers [5]–[8]. It also makes it easier to support multiple users [9]–[11]. Most prior work though assumed narrowband operation.

Frequency selective beamforming that adaptively changes beam patterns in the frequency domain will be required in wideband mmWave systems for two reasons. One reason is the possibility that the best beam pattern varies over frequency in wideband channels due to delay spread [12]. The other reason is that supporting multiple users who want to receive short packets will be inevitable in practical systems [13]. Since many hybrid precoding techniques as well as analog beamforming cannot support frequency selective precoding, only one user with a short packet can receive data at each transmit time interval, which results in a waste of frequency resources. Therefore, it is essential to simultaneously allocate multiple users to different frequencies with their own beam.

In this paper, we develop a hybrid precoding architecture that allocates multiple users to different frequencies with different preferable beamforming vectors in a downlink orthogonal frequency division multiple access (OFDMA) system. The digital precoding part is performed in the frequency domain prior to inverse fast Fourier transform (IFFT) processing; the analog precoding part in the time domain after IFFT processing. We propose a criterion for designing the frequency selective hybrid precoding, which aims for maximizing a geometric mean of beamforming gain per each subband allocated to each user. Since solving directly is difficult, we also propose a near-optimal algorithm to design analog and digital beamforming by using an iterative method. We analyze the theoretical upper bound on the relative beamforming gain compared to the ideal beamforming case where every subband is assumed to use its own best beam pattern. We show that the performance of the proposed algorithm approaches the upper bound closely. Finally, we present simulation results showing the effectiveness of the proposed frequency selective hybrid precoding in the environment where multiple users with heterogeneous traffic coexist.

Throughout this paper, the following notation is used.\(A\) denotes a matrix and \(a\) denotes a vector, \((\cdot)^T\) transpose, and \((\cdot)^*\) conjugate transpose. \(|a|\) is a modulus of \(a\), \(|a|\) is an \(l_2\)-norm of \(a\), and \(\|A\|_F\) is a Frobenius norm of \(A\). \([A]_{i,j}\) denotes \((i,j)\)-th element of \(A\) and \([a]_i\) denotes \(i\)-th element of \(a\). \(\text{Tr}(A)\) is a trace function of \(A\), and \(\prod\text{diag}(A)\) represents the product of the diagonal elements of \(A\).
Consider a downlink OFDMA system where a base station has $N_{TX}$ antennas and $N_{RF}$ RF chains. In each transmit interval, $N_U$ users are assigned. Each user is assumed to receive a single data stream. Throughout this paper, a subband where multiple users are simultaneously allocated to different subbands.

As shown in Fig. 1, frequency selective hybrid precoding consists of two steps: baseband digital precoding performed before the IFFT and RF analog precoding after the IFFT. The $N_{TX} \times N_{RF}$ RF precoding matrix $F_{RF}$ whose elements have a constant modulus is common for all users, and each user $u$ has its dedicated $N_{RF} \times 1$ baseband precoding vector $f_{BB,u}$. In this paper, each user is assumed to have one best beamforming vector for simplicity of notation. We regard this as an ideal beamforming vector in comparison to the practical beamforming vector with a limited number of RF chains.

$$f_{ideal,u} = \frac{1}{\sqrt{N_{TX}}} \begin{bmatrix} e^{2 \pi i k_u} \cdot \frac{1}{N_{code}} & \cdots & e^{2 \pi i (N_{TX} - 1) k_u} \cdot \frac{1}{N_{code}} \end{bmatrix}^T,$$  

where $k_u$ is the selected beam index for user $u$. $N_{code}$ is the number of beamforming vectors used in the beam training phase. A large $N_{code}$ causes longer training time while making the resolution of the beam direction higher. For most of this paper, we assume that $N_{code}$ is the same as $N_{TX}$. The case when $N_{code}$ is larger than $N_{TX}$ will be discussed in Section IV.

Note that we focus on frequency selective beamforming in this paper. The baseline is the beam steering case where there is no limitation on the number of RF chains. Given a limited number of RF chains, there are fewer degrees of freedom. While other work [5]–[11] dedicates these degrees of freedom to supporting spatial multiplexing, our work aims at dedicating them to supporting frequency selective precoding. Frequency selective hybrid precoding combined with spatial multiplexing by distributing limited degrees of freedom is a good topic of future work.

### III. Optimal Criterion for Frequency Selective Hybrid Precoding

The goal of our hybrid precoding design is to make each user’s hybrid precoding vector similar to the ideal beamforming vector as much as possible. Two problems arise: how to define the similarity of the two vectors and how to design the similar vector according to the definition. In this section, we focus on the first problem and define a proper optimality criterion.

To formulate the problem, let us represent the ideal beamforming vectors and the baseband precoding vectors in matrix form

$$F_{ideal} = \begin{bmatrix} f_{ideal,1} & \cdots & f_{ideal,N_U} \end{bmatrix},$$

$$F_{BB} = \begin{bmatrix} f_{BB,1} & \cdots & f_{BB,N_U} \end{bmatrix}.$$  

The overall hybrid precoding matrix can be represented as

$$F_{hybrid} = \begin{bmatrix} f_{hybrid,1} & \cdots & f_{hybrid,N_U} \end{bmatrix} = \begin{bmatrix} F_{RF} f_{BB,1} & \cdots & F_{RF} f_{BB,N_U} \end{bmatrix}.$$  

We assume that the hybrid precoding vector of each user $u$ is normalized such that

$$|f_{hybrid,u}| = |F_{RF} f_{BB,u}| \leq 1, \forall u,$$  

as the ideal beamforming vector, $f_{ideal,u}$. If the hybrid precoding matrix $F_{hybrid}$ is identical to $F_{ideal}$, there will be no loss from hybrid precoding.

First, let us consider the case when $N_U \leq N_{RF}$. There is a trivial solution in this case: $F_{RF}$ is composed of $N_U$ ideal precoding vectors and $F_{BB}$ is a one-to-one mapping matrix whose elements are either 0 or 1. In this case, there is no loss and we can make the precoding matrix $F_{RF} F_{BB}$ exactly the same as the ideal matrix $F_{ideal}$.

A problem arises when $N_U > N_{RF}$. This situation may seem odd because general narrowband MU-MIMO hybrid precoding schemes have the constraint of $N_U \leq N_{RF}$, where $N_U$ means the number of users who can simultaneously
transmit data through the same frequency band. The maximum number of supportable users is limited to $N_{\text{RF}}$ because the rank of the overall hybrid precoding matrix, $\text{rank}(F_{\text{RF}}F_{\text{BB}})$, is limited to $\min(N_{\text{U}}, N_{\text{RF}})$ when $N_{\text{TX}}$ is large. Therefore, the case of $N_{\text{U}} > N_{\text{RF}}$ is impossible in narrowband MU-MIMO hybrid precoding. In contrast, frequency selective hybrid precoding can support more than $N_{\text{RF}}$ users. The main difference comes from the fact that there is no interference among other users when users are allocated to different subbands. While narrowband MU-MIMO hybrid precoding tries to cancel up to $(N_{\text{RF}}-1)$ interferers with $N_{\text{RF}}$ degrees of freedom, frequency selective hybrid precoding can dedicate those degrees of freedom to supporting more than $N_{\text{RF}}$ users.

Although it is possible to support more than $N_{\text{RF}}$ users, this will cause some performance loss compared to the ideal case. To quantify this loss, we define the relative beamforming gain per user compared to the ideal case as

$$G(u) = |f_{\text{ideal},u}^*F_{\text{RF}}f_{\text{BB},u}|^2.$$  

Note that $G(u) \leq 1$ due to (4) and the equality holds if the hybrid precoding vector is the same as the ideal beamforming vector.

We now consider how to define the similarity of two matrices. One frequently used measure is a Frobenius norm of an error matrix that represents the difference between two matrices [5]. By this measure, the optimal criterion can be written as

$$\arg\min_{F_{\text{RF}},F_{\text{BB}}} \frac{||F_{\text{ideal}} - F_{\text{RF}}F_{\text{BB}}||_F^2}{\sum_{n,r} G(u)}$$  

s.t. $|F_{\text{RF}}f_{\text{BB},u}|^2 \leq 1, \forall u$, and $||F_{\text{RF}}||_{n,r} \leq 1, \forall n, r$.  

(6)

Note that the columns of $F_{\text{ideal}}$ are orthonormal to each other and $\text{rank}(F_{\text{RF}}F_{\text{BB}}) \leq N_{\text{RF}}$. These differences from [5] require a different approach. In particular, the fact that $F_{\text{ideal}}$ has orthonormal column vectors whose elements have constant modulus makes this problem simple. The solution can be derived by the Eckart-Young-Mirsky theorem [15] as

$$F_{\text{RF}} = \sqrt{N_{\text{TX}}} U_{N_{\text{RF}}}, \quad F_{\text{BB}} = \frac{1}{\sqrt{N_{\text{TX}}}} U_{N_{\text{RF}}}^* F_{\text{ideal}},$$  

(7)

where $U_{N_{\text{RF}}}$ is a matrix composed of dominant $N_{\text{RF}}$ left eigenvector of $F_{\text{ideal}}$. We can find $U_{N_{\text{RF}}}$ that is a unitary matrix whose elements have a constant modulus due to the characteristics of $F_{\text{ideal}}$.

Before examining the physical meaning of (6), we introduce another approach and compare it with (6). Let us regard $f_{\text{ideal},u}$ as each user’s $1 \times N_{\text{TX}}$ virtual channel vector, which represents the channel direction in its dominant eigenmode. Then, $f_{\text{ideal},u}^* F_{\text{RF}}$ is regarded as an $1 \times N_{\text{RF}}$ effective virtual channel from RF chains to user $u$. In this case, we can consider a two-step approach. The first step is to find $F_{\text{RF}}$ maximizing the sum of $|f_{\text{ideal},u}^* F_{\text{RF}}|^2$, while the second step is to design $F_{\text{BB}}$ such that each columns of $F_{\text{BB}}$ is identical to the maximum ratio transmission beamforming associated with $f_{\text{ideal},u}^* F_{\text{RF}}$. In matrix form, the first step can be written as

$$\arg\max_{F_{\text{RF}}} \text{Tr}(F_{\text{ideal}}^* F_{\text{RF}} F_{\text{RF}}^* F_{\text{ideal}})$$  

s.t. $||F_{\text{RF}}||_{n,r} = 1, \forall n, r$.  

(8)

The second step is

$$f_{\text{BB},u} = \alpha_u (f_{\text{ideal},u}^* F_{\text{RF}})^*, \forall u$$  

(9)

where $\alpha_u$ is a normalization factor to meet the power constraint and can be calculated as $\alpha_u = 1/||F_{\text{RF}}F_{\text{RF}}^* f_{\text{ideal},u}||$.

Interestingly, the solution to this problem is the same as the solution to (6). By using the definition of the relative beamforming gain in (5), the optimal criterion (8) combined with (9) can be rewritten as

$$\arg\max_{F_{\text{RF}},F_{\text{BB}}} \sum_{u=1}^{N_{\text{U}}} G(u).$$  

(10)

This indicates that the goal of these two different optimal criteria is to maximize the sum of the relative beamforming gain. This strategy seems reasonable at first glance, but it is not suitable for our hybrid precoding design in mmWave systems. Since the columns of $F_{\text{RF}}$ are orthogonal, the optimal solution is the same as selecting only $N_{\text{RF}}$ users among $N_{\text{U}}$ users while making other users’ beamforming gain zero. Therefore, it cannot meet our initial target to support more than $N_{\text{RF}}$ users with acceptable loss to fully use frequency resources.

Instead of maximizing the sum of the relative beamforming gain, we propose to maximize the product of the relative beamforming gain as

$$\arg\max_{F_{\text{RF}},F_{\text{BB}}} \prod_{u=1}^{N_{\text{U}}} G(u).$$  

(11)

In other words, we use the geometric mean of $G(u)$ instead of the arithmetic mean of $G(u)$ as an objective function [16]. The rationale of (11) is to equally distribute the relative loss compared to the ideal beamforming among all users. If the ideal beamforming is used, $G(u)$ will become one for all users. This does not mean that the ideal beamforming makes all users’ SNR or capacity identical. Apart from beamforming, rate control can be performed according to each user’s actual channel SNR. Our target is to make each user’s relative loss, from the viewpoint of beamforming gain, both small and fair. The revised criterion is

$$\arg\max_{F_{\text{RF}}} \prod_{u=1}^{N_{\text{U}}} \text{diag}(F_{\text{ideal}}^* F_{\text{RF}} F_{\text{RF}}^* F_{\text{ideal}})$$  

s.t. $||F_{\text{RF}}||_{n,r} = 1, \forall n, r$.  

(12)

After finding $F_{\text{RF}}$, we can find $F_{\text{BB}}$ from (9). Unfortunately, the objective function in (12) is a nonlinear function, and we are not aware of a closed-form solution. In the next section, we propose an iterative method to obtain a near-optimal solution and examine the maximum value of $G(u)$ that this solution can achieve.
Algorithm 1 Finding a hybrid precoding matrix maximizing the geometric mean of the relative beamforming gain.

Input: $\mathbf{F}_{ideal}$
$\mathbf{U}_{N_{RF}}$ : Dominant $N_{RF}$ left eigenvectors of $\mathbf{F}_{ideal}$
$[\mathbf{F}_{RF}]_{n,r} = e^{j\varphi_n,r}$, $\forall n, r$
$\mu = \prod \text{diag}(\mathbf{F}_{ideal}\\mathbf{F}_{RF}^*\mathbf{F}_{ideal})$
$C_{stop} = 0$
while $C_{stop} = 0$ do
  $C_{stop} = 1$
  for $n = 1 : N_{TX}$ do
    for $r = 1 : N_{RF}$ do
      $G = \mathbf{F}_{ideal}^*\mathbf{F}_{RF} = [\mathbf{g}_1^T \cdots \mathbf{g}_N^T]^T$
      $d = \sum_{n=1}^{N_U} \frac{[f_{ideal,u}_n]^2 [f_{RF,r}_n]^2}{|g_n|^2}$
      $\hat{\mathbf{F}}_{RF} = \mathbf{F}_{RF}$
      $[\mathbf{F}_{RF}]_{n,r} = \frac{\mathbf{g}_n}{|\mathbf{g}_n|}$
      $\hat{\mu} = \prod \text{diag}(\mathbf{F}_{ideal}\\hat{\mathbf{F}}_{RF}^*\hat{\mathbf{F}}_{RF}^*\mathbf{F}_{ideal})$
      if $\hat{\mu} > \mu$ then
        $\mu = \hat{\mu}$
        $\mathbf{F}_{RF} = \hat{\mathbf{F}}_{RF}$
        $C_{stop} = 0$
      end if
    end for
  end for
end while
$f_{BB,u} = \frac{f_{BB,1}^* f_{ideal,u}^*}{|f_{BB,1}^* f_{ideal,u}^*|}, \forall u$
$\mathbf{F}_{BB} = [f_{BB,1}^* \cdots f_{BB,N_{TX}}^*]$.

Output: $\mathbf{F}_{RF}, \mathbf{F}_{BB}$

IV. HYBRID PRECODING MAXIMIZING THE GEOMETRIC MEAN OF THE RELATIVE BEAMFORMING GAIN

Considering that the elements of $\mathbf{F}_{ideal}$ and $\mathbf{F}_{RF}$ have the same modulus, the elements can be represented as

$$[\mathbf{F}_{ideal}]_{n,u} = \frac{1}{\sqrt{N_{TX}}} e^{j\theta_{n,u}}, \quad [\mathbf{F}_{RF}]_{n,r} = e^{j\varphi_{n,r}}. \quad (13)$$

The optimization problem (12) is identical to

$$\arg\max_{\mathbf{F}_{RF}} \sum_{u=1}^{N_U} \log(|f_{ideal,u}^*\mathbf{F}_{RF}|^2) \quad (14)$$

subject to $[\mathbf{F}_{RF}]_{n,r} = 1, \forall n, r$.

This can be rewritten as

$$\arg\max_{\Phi} g(\Phi), \quad (15)$$

where $\Phi$ is an $N_{TX} \times N_{RF}$ matrix whose $(n,r)$-th element is $\phi_{n,r}$, and

$$g(\Phi) = f(\mathbf{F}_{RF}) = \sum_{u=1}^{N_U} \log(N_{TX}|f_{ideal,u}^*\mathbf{F}_{RF}|^2). \quad (16)$$

By using $\Phi$, we can eliminate the constraint in (14).

The partial derivative of $g(\Phi)$ with respect to the variable $\phi_{n,r}$ can be calculated as

$$\frac{\partial g(\Phi)}{\partial \phi_{n,r}} = \frac{N_{RX}}{\sum_{u=1}^{N_U} \log(\sum_{r=1}^{N_{RF}} e^{j\phi_{n,r}})} \left( \sum_{n=1}^{N_{TX}} \sum_{r=1}^{N_{RF}} e^{j\phi_{n,r}} e^{j\theta_{n,u}} \right). \quad (17)$$

where $\Re(\cdot)$ represents the imaginary part of a complex number. We can rewrite (17) with $\mathbf{F}_{RF}$ and $\mathbf{F}_{ideal}$ as

$$\frac{\partial g(\Phi)}{\partial \phi_{n,r}} = \frac{N_{RX}}{\sum_{u=1}^{N_U} \log(\sum_{r=1}^{N_{RF}} |\mathbf{F}_{ideal,u}^*\mathbf{F}_{RF}|^2)} \left( \sum_{n=1}^{N_{TX}} \sum_{r=1}^{N_{RF}} |\mathbf{F}_{ideal,u}^*\mathbf{F}_{RF}|^2 \right). \quad (18)$$

It is difficult to find a closed-form solution to satisfy the equation $\frac{\partial g(\Phi)}{\partial \phi_{n,r}} = 0$ for all users. Instead, we propose an iterative algorithm to find a solution in Algorithm 1. In the algorithm, $\text{sgn}(\cdot)$ is the sign function, $\angle(\cdot)$ is the phase function, and $\Delta$ is a quantization level of a RF phase shifter, which is assumed to be the same as the quantization level of $\mathbf{F}_{ideal}$. In this paper, we focus on finding a near-optimal value and do not consider the reduction in complexity.

Algorithm 1 aims at the equal and high beamforming gain for all users. How much loss is expected? We do not know the exact optimal loss because the global optimum is not known. Fortunately, an upper bound of the relative beamforming gain can be derived when $\mathbf{F}_{ideal}$ consists of orthonormal vectors. In this case, the per-user relative beamforming gain satisfies

$$\left( \prod_{u=1}^{N_U} G(u) \right)^{1/N_U} \leq \frac{1}{N_U} \sum_{u=1}^{N_U} G(u) \leq \frac{\min(N_{RF},N_U)}{N_U}. \quad (19)$$

The first inequality comes from the fact that the geometric mean is always smaller than the arithmetic mean; equality holds if all users have the same relative beamforming gain. The second inequality comes from (10); the equality holds...
if the optimal solution (7) for maximizing the arithmetic mean is applied. This shows that the relative beamforming gain compared to the ideal beamforming is upper-bounded by $10 \log_{10}(\frac{\sum_{u} (\text{SNR}_{\text{ideal}}(u))}{\text{SNR}_{\text{ideal}}(u_{\text{max}})})$ dB. This provides insight about the trade-off between the number of users and beamforming gain. If there are many short-packet users and the bandwidth is underutilized, it is beneficial to allocate more users with some loss in beamforming gain by exploiting more bandwidth. Alternatively, if many users have heavy traffic data and thus bandwidth is already fully utilized, transmitting one user at a time without beamforming loss will be the best strategy.

In Section II, we assumed that the number of beam steering vectors $N_{\text{code}}$ is less than or equal to $N_{\text{TX}}$. The ideal beamforming vectors in $\mathbf{F}_{\text{ideal}}$, therefore, are orthonormal. This results in the simple solution (7) to maximizing the arithmetic mean. The solution in the case when $N_{\text{code}} > N_{\text{TX}}$, however, is not known because orthonormality is not guaranteed any more. In the proposed algorithm, maximizing the geometric mean can be applied to $N_{\text{code}} > N_{\text{TX}}$ without any modification because it does not depend on orthonormality. Even though the case when $N_{\text{code}} = N_{\text{TX}}$ provides insight that the general trend will not change in other cases, the exact comparison between the two criterion, however, will be necessary. This remains as future work.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed algorithm and how close it can approach the upper bound. Also, we show how it works in a heterogeneous situation where users coexist with different types of traffic: finite and infinite buffer size.

Fig. 2 shows the relative beamforming gain $G(u)$ according to the number of users $N_{\text{U}}$ when a base station has 16 antennas and 4 RF chains. The quantization resolution $\Delta$ in Algorithm 1 is set to $\frac{2\pi}{N_{\text{TX}}}$, which is the same as that of $\mathbf{F}_{\text{ideal}}$. The worst scenario, where all $N_{\text{U}}$ users have different ideal beamforming vectors, is assumed, and the simulation is performed over all possible combinations $\binom{N_{\text{TX}}}{N_{\text{U}}}$ at each $N_{\text{U}}$. In Fig. 2, we can see that the geometric mean of $G(u)$ is similar to the arithmetic mean, which suggests the loss is equally divided over all users by the proposed algorithm.

To show the effectiveness of the proposed hybrid precoding design in a practical heterogeneous traffic scenario, a simple simulation is performed. In this scenario, there are multiple users who want to receive real-time traffic with finite small buffer size as well as users who want to receive best-effort traffic with a nearly infinite buffer size. Since scheduling is not within the scope of this paper, a single best-effort user with infinite buffer size is assumed. The system bandwidth is 500MHz, $N_{\text{TX}}$ is 16, and $N_{\text{RF}}$ is either 4 or 8. The number of real-time users at every transmit time interval is uniformly distributed over $[0 \sim N_{\text{U,max}}]$ and their packet size is associated with the rate 5Mbps. Each user acquires a preferable beamforming vector at initial beam training phase and each user’s SNR ideal which includes its beamforming gain through the ideal beamforming is uniformly distributed over $[0 \sim 25]$dB. We assume that the real-time users are first assigned with higher priority than the best-effort user. The user $u$’s rate $R(u)$ is calculated from

$$R(u) = B(u) \log_{2}(1 + \frac{\text{SNR}_{\text{ideal}}(u)}{10} |\mathbf{f}_{\text{ideal},u}^{T} \mathbf{F}_{\text{RF}} \mathbf{f}_{\text{BB},u}|^2),$$

where $B(u)$ is the bandwidth allocated to user $u$. Note that the base station needs to allocate the bandwidth to each user subject to $\sum B(u) \leq 500$MHz.

Fig. 3 shows the best-effort traffic user’s rate according to $N_{\text{U,max}}$. Analog beamforming has the poorest performance because it can assign only one user at a time and thus the best-effort traffic user cannot be assigned if a real-time user exists at that time. Hybrid precoding with the criterion maximizing arithmetic beamforming gain selects only $N_{\text{RF}}$ beams. Since

Fig. 3. Rate of a best-effort user versus the maximum number of real-time users at each transmit time interval.

Fig. 4. Total bandwidth usage ratio versus the maximum number of real-time users at each transmit time interval.
it selects dominant eigenvectors of the matrix consisting of all users’ preferred beamforming vectors, it first selects the beams that many users share and thus it is possible to support more than $N_{RF}$ users in that case. Its performance, however, is still lower than that of the proposed algorithm using the criterion maximizing geometric beamforming gain. The performance gap between the two hybrid precoding techniques with different criterion gets larger as $N_{RF}$ is smaller.

The rate gap mainly results from the difference between bandwidth usage, as shown in Fig. 4. We can see that the spectral efficiency of the proposed algorithm is lower than that of hybrid beamforming maximizing the arithmetic mean due to loss in beamforming gain. Even though the latter has a higher spectral efficiency, it cannot fully utilize the overall bandwidth, which results in a waste of frequency resources.

We verify the advantage of the proposed algorithm from the view of real-time users. Fig. 5 shows the average number of unassigned real-time traffic users at each transmit time interval. Since hybrid precoding maximizing the arithmetic mean and analog beamforming cannot allocate many users at one time, the number of unassigned users is large. This will result in an increase in latency.

VI. Conclusions

In this paper, we proposed a frequency selective hybrid precoding technique in mmWave OFDMA systems, in which analog precoding is performed in the time domain and digital precoding is performed in the frequency domain. Given that a base station with a small number of RF chains needs to allocate multiple subbands to multiple users with their own preferable beamforming vectors, we suggested a criterion of maximizing the geometric mean of the relative beamforming gain per user. The solution satisfying this optimal criterion minimizes the reduction in beamforming gain compared to the ideal beamforming and also equalizes the reduction experienced by each user. We showed that the theoretical upper bound on the relative beamforming gain compared to the ideal beamforming.

When this optimal criterion is applied. We also proposed the iterative algorithm to find a near-optimal solution to this non-linear optimization problem. Simulation results illustrated that frequency selective precoding is essential in the environment where many users with small packets exist. The results also showed that the optimal criterion for maximizing the geometric mean is more desirable than the optimal criterion for maximizing the arithmetic mean.

ACKNOWLEDGEMENT

This research was supported by a gift from Huawei Technologies Co. Ltd.

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