Radio Map Recovery and Noise Reduction Method for Green WiFi Indoor Positioning System Based on Inexact Augmented Lagrange Multiplier Algorithm

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Abstract—Currently, WiFi indoor positioning system based on IEEE 802.11 is widely attractive for its free infrastructure and high localization performance. However, due to working on-demand strategy in green WiFi scenario, the access points are not always available for mobile when radio map is built in the offline phase. Radio map with unknown received signal strength is not valid for positioning and usually be replaced by the minimum value, which leads to poor positioning performance. In this paper we propose a radio map recovery method based on inexact augmented Lagrange multiplier (IALM) algorithm, which achieves to precisely recover the missing received signal strength in the radio map for those access points unavailable in the offline. By solving the nuclear norm minimization, the IALM algorithm could not only recover the missing received signal strength, but also reduce the noise effectively. We have implemented the proposed method in our lab and evaluated its performances. The experiment results indicate the proposed method could precisely recover the radio map and achieve good positioning performance.

Keywords—WiFi; indoor positioning; radio map; IALM

I. INTRODUCTION

Nowadays, location based service plays a more and more important role in various indoor applications, such as shopping, touring, first-aid and etc. [1]. Though the world's most widely employed satellite navigation GPS is successfully applied in many fields, its signal is seriously blocked by walls and roofs for indoor environment. Hence, indoor positioning service is in great need. In the wake of development in the Internet, WiFi is popularized as the information infrastructure throughout the world. Millions of access points (APs) are deployed, and people could freely access Internet at any time. In the meanwhile, due to its ubiquitous network architecture and no additional hardware requirements, WiFi indoor positioning system based on the received signal strength (RSS) has actually become the first option for indoor location based service, such as in shopping malls, offices and airports [2].

Generally, there are two types of RSS-based localization models for WiFi indoor positioning systems: triangle method and fingerprint method [3]. Triangle method requires at least three known AP locations and then measures their distances to the mobile for further estimating the mobile position. However, in practice, the access point locations are not always available. In addition, when the signal propagation path loss and shadow fading in indoor environment are taken into consideration, the inaccurate distance measurement between AP and mobile would inevitably hinder the estimation for mobile localization. For the fingerprint method, the position estimation depends only on the RSS characterization. Typically, there are two stages for the fingerprint method: offline phase and online phase [7]. A pre-built RSS data base, called radio map, is constructed in the offline phase, which plays a key link in WiFi positioning system. In the online phase, the mobile location is determined by comparing the Euclidean distances in RSS space between radio map and online RSS, which employs the nearest neighbor (NN) or the k nearest neighbor (KNN) algorithm. Because AP location and its distance to the mobile are never the prerequisites, fingerprint method has actually become the first option for the RSS based indoor positioning technique in the traditional WiFi indoor positioning system.

However, in green WiFi scenario, APs would be powered off randomly according to the data communication demanding by the working on-demand strategy [4]. RSS missing from unavailable access points would lead to incomplete radio map, which means some RSS are unknown when radio map is built. On the other hand, RSS would also be corrupted by noises generated from mobile diversity, complexity environment (temperature, humidity) and indoor multipath (human walking, walls blocking, doors and windows opening randomly). All these problems are obviously great challenges to green WiFi indoor positioning system.

Recently, with compressed sensing making great progress in theory and application, low rank matrix sparse representation has become an effective and popular way for data processing [7]. There exist several matrix recovery algorithms[8,9], which aim to recover an unknown matrix when only a fraction of its entries are known, such as iterative threshold algorithm [10], accelerated proximal gradient (APG) [11], dual algorithm [12], singular value thresholding (SVT) [13,14] and inexact augmented Lagrange multiplier (IALM) [15].

In particular, the IALM algorithm outperforms others in effective computational cost and easy implementation. IALM has the merits of less iterations and adaptation for higher dimension matrix. It considers the matrix recovery as a convex relaxation of a rank minimization problem, and approximates the matrix with minimum nuclear norm among all matrices obeying a set of convex constraints. Therefore, in this paper,

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II. WIFI INDOOR POSITIONING SYSTEM

A. Offline Phase: Radio map Construction

In the offline phase, a database called radio map is pre-built. It includes plenty of reference points, whose locations and corresponding RSS are recorded. Radio map plays a significant role in the fingerprint method. It contains two parts of information: reference points location and RSS. Assuming there are \( m \) reference points and \( n \) APs, and \( L \in \mathbb{R}^{m \times n} \) be these reference points’ locations:

\[
L = [(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)]^T
\]

And let \( \Psi \in \mathbb{R}^{m \times n} \) be the RSS of reference points:

\[
\Psi = \begin{bmatrix}
\psi_{11} & \psi_{12} & \cdots & \psi_{1n} \\
\psi_{21} & \psi_{22} & \cdots & \psi_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\psi_{m1} & \psi_{m2} & \cdots & \psi_{mn}
\end{bmatrix}
\]

where \( \psi \) is the average of RSS readings at the \( i \) th reference point from the \( j \) th AP. Then radio map is expressed as:

\[
\text{RadioMap} = [L, \Psi]
\]

where \((\cdot)^T\) is the matrix transposition operator. Basically, RSS \( \Psi \) is the most vulnerable part in radio map. So here and in the sequel, we address RSS \( \Psi \) as radio map for convenience.

During radio map construction in green WiFi, APs will be powered off randomly due to data communication demand for energy efficiency, so that mobile might not sense any RSS from some access points at particular time. One way to complete the radio map is to assume the mobile is out of the AP signal coverage and fill the RSS manually with some value (e.g., -100dBm), which is the signal floor of RSS. However, this way will no doubt lead to poor positioning accuracy, especially for those APs close to the mobile but being powered off for energy efficiency. It is highly possible that in the online phase these APs are powered on and mobile could sense strong RSS. The difference between RSS in the offline phase and online phase will finally achieve poor positioning performance.

B. Online Phase: KNN Algorithm

In online phase, RSS \( \tilde{\psi} \in \mathbb{R}^{m \times n} \) is received by mobile, and comparison between online \( \tilde{\psi} \) and offline \( \Psi \) is implemented to estimate the mobile location from the reference points locations \( L \). The most popular algorithm is the KNN, whose estimation is based on the Euclidean distance in RSS:

\[
d_j = \| \psi_j - \tilde{\psi} \|_2 \quad \forall j = 1, \ldots, m
\]

where \( \| \cdot \|_2 \) is the Euclidean distance, and \( \psi_j \) is the column of \( \Psi \).

Then KNN chooses \( K(>1) \) reference points with the shortest Euclidean distance \( d_j \) as the mobile candidate positions. The mobile position \((\hat{x}, \hat{y})\) is finally estimated by averaging these reference point locations as follows:

\[
(\hat{x}, \hat{y}) = \frac{1}{K} \sum_{i=1}^{K} (x_i, y_i)
\]

In conclusion, we could see that radio map plays a crucial role in the WiFi indoor positioning system. If RSS \( \Psi \) is not accurate or even fault in the radio map, the position results will be definitely poor. Therefore, we should carefully recover radio map to assist the mobile to make proper position estimation.

III. RADIO MAP RECOVERY AND DENOISING IN GREEN WIFI

A. Matrix Recovery Overview

Matrix recovery is a way to solve the \( l_1 \) norm optimization problem and recover low rank matrix. Its mathematic model could be expressed as:

\[
\min \| A \|_1
\]

For green WiFi, due to not all the RSS are available when building radio map, we should recover these missing RSS by matrix recovery. The radio map recovery is modeled as follows:

\[
P_\Omega(A) = P_\Omega(D)
\]

where \( A \in \mathbb{R}^{m \times n} \) is the desired radio map RSS \( \Psi \) as shown in Eq. (2) with all APs powered on, and \( D \) is the actual radio map we could get. The operator \( P_\Omega : \mathbb{R}^{m \times n} \rightarrow \mathbb{R}^{m \times n} \) is a linear projection operator, which incompletes radio map and fills those unavailable APs with the minimum RSS value:

\[
P_\Omega(D) = \begin{bmatrix}
D_{ij} & (i, j) \in \Omega \\
-100\text{dBm} & (i, j) \notin \Omega
\end{bmatrix}
\]

where \( \Omega \) is the set of RSS that is available at the \( i \) th reference point from the \( j \) th AP.
Unfortunately, Equ. (6) is an NP-hard problem. However, the nuclear norm of the matrix is the convex envelope of the matrix rank, which means it could be relax to nuclear matrix norm. Taking the convex optimization problem as:
\[
\min \| A \|_* \quad \text{s.t. } P_{\Omega}(A) = P_{\Omega}(D) \tag{9}
\]
where \( \| \cdot \|_* \) is the nuclear norm operator (i.e. the sum of its singular values). By solving the convex optimization problem, we are able to get the optimal solution of the above problem and recover the incomplete radio map.

### B. Radio Map Recovery by IALM

The IALM algorithm could efficiently solve the convex optimization problem. For the noise-free case, based on the basis of IALM, radio map recovery could be expressed as:
\[
\min \| A \|_* \quad \text{s.t. } A + E = D, \quad P_{\Omega}(E) = 0, \quad P_{\Omega}(A) = P_{\Omega}(D) \tag{10}
\]
where \( E \) is a temporary matrix for Lagrange operation whose entries are also 0 if \((i, j) \notin \Omega \). So we could formulate the augmented Lagrange function to get the desired radio map \( A \):
\[
L(A, E, Y, \mu) = \| A \|_* + \langle Y, D - A - E \rangle + \frac{\mu}{2} \| D - A - E \|_F^2 \tag{11}
\]
where \( Y \) is a temporary sparse matrix. The operator \( \langle \cdot \rangle \) means:
\[
\langle Y, D - A - E \rangle = \text{trace} \left( Y^T (D - A - E) \right) \tag{12}
\]

The IALM updates \( A, E \) and \( Y \) iteratively to minimize \( L(A, E, Y, \mu) \) with \( D \) fixed. Supposing \( (D - E + \mu^{-1}Y) \) has \( r \) ordered singular values with \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq 0 \):
\[
A_{k+1} = \arg \min_A L = U_k \text{diag} \left( \max(0, \sigma_i - \mu_k) \right) V_k^T \tag{13}
\]
\[
E_{k+1} = P_{\Omega} \left( D - A_{k+1} - Y_k / \mu_k \right) \tag{14}
\]
\[
Y_{k+1} = Y_k + \mu_k \left( D - A_{k+1} - E_{k+1} \right) \tag{15}
\]
where \( U \) and \( V \) are respectively left and right singular value vectors, \( k \) is the iteration step number, \( \mu \) is the relaxation parameter, and we set \( \mu_k = \beta \| D \|_F \) in this paper. Updating \( \mu \) with \( \mu_{k+1} = 1.6 \mu_k \). We can gain the desired radio map until the shrink iteration stops when the computation error is satisfied:
\[
\frac{\| P(M - A_k) \|_F}{\| P(M) \|_F} < \varepsilon \tag{16}
\]
where \( \varepsilon \) is the stopping criteria, and we set \( \varepsilon = 10^{-4} \) in this paper.

### C. Radio Map Noise Reduction by IALM

In practice, during radio map construction, noises coming from mobile diversity, complexity environment and indoor multipath will be introduced to the RSS. The IALM algorithm has good scalability, which could not only achieve radio map recovery but also noise reduction.

For the noisy radio map case, we could model the radio map recovery as the following convex optimization:
\[
\min \| A \|_* + \gamma \| P_{\Omega}(E) \|_F \quad \text{s.t. } A + E = D, \quad P_{\Omega}(A) = P_{\Omega}(D) \tag{17}
\]
Comparing with the Equ. (10), the objective function has an extra noise, where \( \gamma \) is a positive noise weighting parameter. We set \( \gamma \) as a constant equaling to \( (\max(m, n))^2 \). \( E \) is a noisy matrix. Here we formulate the part of the inexact augmented Lagrange function as follows:
\[
L'(A, D, E, Y, \mu) = L(A, E, Y, \mu) + \gamma \| P_{\Omega}(E) \|_F \tag{18}
\]

Similar to the Equ. (11) operation, we implement the shrink iteration for \( A, E \) and \( Y \):
\[
A_{k+1} = \arg \min_A L' = U_k \text{diag} \left( \max(0, \sigma_i - \mu_k^{-1}) \right) V_k^T \tag{19}
\]
\[
E_{k+1} = \frac{\mu}{\mu + 2\gamma} P_{\Omega} \left( D - A + \mu^{-1}Y \right) + P_{\Omega} \left( D - A + \mu^{-1}Y \right) \tag{20}
\]
\[
Y_{k+1} = Y_k + \mu_k \left( D - A_{k+1} - E_{k+1} \right) \tag{21}
\]

When the iteration stop condition meets Equ. (16), we could reduce the noise from the desired radio map \( A \).

### IV. IMPLEMENTTION AND PRFORMANCE ANALYSIS

#### A. Experiment Environment

We use a laptop (Lenovo G470) to collect RSS both in offline phase and online phase. The experiment area is our lab, which is a typical office environment with 27 APs (Linksys WRT54G) setting in IEEE 802.11b/g mode. The floor plan is shown in Fig. 1. The interesting area for localization is illustrated with yellow color.

![Figure 1. Floor plan for indoor localization](image)

In our experiment, we divide the corridor into grids with 0.5 meter apart, which is shown in Fig. 2 with little red crosses as the reference points. In total, we have 823 reference points for the radio map.
We set all the APs powered on and get the desired radio map as Fig. 3 shown. The row in Fig. 3 is the number of the reference points, and the column is the number of APs. The legend bar in colors is in dBm scale, where dark red means RSS is strong, and dark blue means no RSS is sensed from AP.

\[ \text{Figure 3. Desired radio map} \]

B. Radio Map Recovery

On account of noise coming from mobile diversity, complexity environment and indoor multipath, we add additive white Gaussian noise to the desired radio map shown in Fig. 4.

\[ \text{Figure 4. Noisy radio map} \]

In order to simulate the working on-demand strategy in green WiFi, we suppose there are five APs randomly powered off for energy efficiency when we built radio map. Fig. 5 illustrates a noisy radio map with some AP unavailable but filling manually with -100 dBm.

\[ \text{Figure 5. Radio map with five access points powered off randomly} \]

According to IALM algorithm, the radio map shown in Fig. 5 is subject to low rank, so that it could be recovered by the IALM algorithm. And the recovered result is provided in Fig. 6.

\[ \text{Figure 6. Recovered radio map with noise by IALM} \]

Comparing with Fig. 3 and Fig. 6, we could see that the radio map is precisely recovered from Fig. 5. If we minus the recovered radio map from the desired radio map, we get the recovery error which is illustrated in Fig. 7. In this figure, we could see that most of the recovery errors are around ± 3dBm.

\[ \text{Figure 7. Radio map recovery error} \]

In order to test the positioning performance, we compare four scenarios. In all these scenarios, we implement the KNN algorithm based on Equ. (4) and Equ. (5), and suppose the online RSS are free from suffering RSS missing for the brevity of discussion in this paper. And the RSS in the online phase recovery method could be found in Ref. [14].

The first scenario is an ideal situation, which means the desired radio map is built when all APs are powered on as shown in Fig. 3. The second scenario is the radio map with noise as shown in Fig. 4, which comes from the first scenario by adding additive white Gaussian noise to the desired radio map. The third scenario is the radio map based on the second scenario with 5 APs randomly powered off in green WiFi, and its missing RSS are manually filled with -100 dBm as Fig. 5 shown. The fourth scenario is the radio map based on the second scenario with 5 APs randomly powered off but recovered by the proposed method, which is shown in Fig. 6. Fig. 8 provides the details of simulation results.

\[ \text{Figure 8. Positioning performance when 5 APs randomly off} \]
It is concluded that the proposed system makes a very good recovery when some APs are unavailable comparing with the one filled manually with -100 dBm. In the meanwhile, the recovered radio map positioning performance greatly outperforms the noisy radio map as shown in Fig. 5.

It is worth to point out that the IALM algorithm has a better performance than other algorithms in computation errors, such as SVT [14]. Based on Equ. (16), we test the computation error when different APs are closed randomly, which is illustrated in Fig. 9. It indicates that with the increase of powered off APs, both of the two algorithms computation errors gradually boost. But the IALM algorithm is much better than SVT. And when 8 APs are powered off, the computation error of IALM is as low as 0.05, comparing with SVT algorithm having computation error about 0.22, which could obviously verify the IALM algorithm has a better performance.

On the other hand, the computation error is only one of the factors that affect the recovered radio map performance. Noises are also another important factor that should be considered. In order to test the proposed method, we provide cumulative probability at 2 meter positioning error for the recovered radio map under different conditions of noise and powered-off APs, which is shown in Fig. 10. Based on Fig. 10, it is concluded that more accurate positioning performance in green WiFi requires less APs being powered off, which also meets our common knowledge and experience.

In the whole, IALM algorithm can achieve low rank matrix recovery and satisfy the demands of the indoor positioning in green WiFi. We could clearly see from these figures above that radio map plays a core role in the WiFi indoor positioning system. If the radio map is not well build or recovered in the offline, the positioning performance will be seriously degraded.

V. CONCLUSION

Due to the green WiFi working on-demand strategy, APs are not always powered on, which leads to great challenges to the WiFi indoor positioning system. In this paper, we propose a radio map recovery and noise reduction method for green WiFi indoor positioning system based on IALM algorithm. We test the proposed method in a typical office environment, and the experiment results show the recovered radio map has good performance in positioning and noise reduction. In addition, comparing with other matrix recovery algorithm, IALM is good at the computation error.

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